1st ICA-JAPAN TRAINING COURSE
ON
ENHANCEMENT OF FARMERS' INCOME & POVERTY REDUCTION THROUGH COOPERATIVES

January 2008

MODULE: FINANCIAL MANAGEMENT

Faculty: Prof. Shiladitya Roy

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January 25, 2008

To : The Participants of ICA Programme

As you are aware, the sessions on Financial Management are scheduled to commence from the 1st February, 2008. For these sessions, the following material are enclosed:

As a prelude to the discussions on 1st February, it would be appreciated if you could please go through the material on Working Capital.

Please remember to bring with you this material set which would be required for discussions.

With best wishes and looking forward to seeing you.

Shiladitya Roy

Encl: as above

CONCEPTS OF WORKING CAPITAL*

INTRODUCTION

Most income generating projects entail investments in assets the proper use of which is supposed to generate a surplus leading to income for the beneficiaries. The assets are broadly of two types – one consisting of equipment, plant, machinery, infrastructure, etc., and these are variously titled as Fixed Assets or Capital Assets. The other type of assets are of a circulating nature like raw material, semi-finished products or work-in-process, finished products and dues from purchasers of the goods or services - variously called debtors or accounts receivables and, some cash balance for contingencies. These type of assets which are basically of a circulating nature are referred to as working capital. This note provides a conceptual and a practical framework for comprehending the notion of working capital.

A TECHNICAL DESCRIPTION OF WORKING CAPITAL

There are two technical terms associated with working capital – gross working capital and net working capital. Gross working capital is the total of current assets like stocks, debtors, cash. Net working capital is the differences between gross working capital or current assets and current liabilities. In effect, net working capital represents that part of the gross working capital or current assets that are met through long term sources of funds.

A PRACTICAL UNDERSTANDING TO WORKING CAPITAL

The concept of working capital can be best understood through relating to a practical production – oriented setting, which is pictured below.

Put simply, working capital is the capital with which we carry on our work.

Now, what is the work in a production setting?

Procurement or Purchase of raw material – processing into finished product – Marketing and getting the sales proceeds.

Therefore, what happens in this process:

- One spends money in acquiring stock or inventory of raw material and as long as these are not sold - the money spent on the stock remaining is blocked.

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* Teaching Note prepared by Prof. Shiladitya Roy of the Institute of Rural Management, Anand.

One starts using the material to process or manufacture the final product – this is work-in-process-consists of money blocked in raw material and partly in processing or manufacturing costs.

The work-in-process gets converted to finished goods – ready for sale. However, till they are sold – they represent money blocked in raw material and manufacturing costs.

Now one sells the finished good. If one gets cash, well and good and if one gives credit – then the money gets blocked again on Debtors or A/c Receivables.

Thus one may have money blocked in Raw Material, Work-in-process, Finished Goods and Debtors and Cash which are called Current Assets and in total represent:

GROSS WORKING CAPITAL

Suppose one gets Credit (i.e., can pay later) on material purchase, and/or on expenses like Salary, Electricity, Fuel etc.,

These type of liabilities called Current Liabilities reduce the amount of funds blocked in Gross Working Capital.

Therefore,

Current Assets – Current Liabilities = Net Working Capital

The Importance of Considering Working Capital

As would be apparent from its very nature, working capital is the life blood of an enterprise which enables it to perform and continue its operations. However, quite often in a project formulation stage, the entire emphasis is placed on the Fixed Asset planning – ignoring or unconsciously down playing the requirement of working capital. This leads to choking of the operations of the enterprise, leading to a vicious circle of inadequate working capital – low activity level below break-even – losses – leading to further reduction in working capital. At the other extreme can be the case of an enterprise having too much working capital, thus locking in unnecessarily larger amount of funds – leading to reduced margins. In view of this, it is of utmost importance to properly assess the working capital requirements of an enterprise and to ensure the corresponding availability of funds.

The substance of working capital can be further reinforced by working on a couple of exercises which follow.

Exercise-1 on Working Capital

A Company say 'A' Co., plans to produce 600 units of furniture. The selling price of a unit is Rs.500/-. The expected ratios of cost to selling price are raw materials 60%, direct wages 10%, production overheads 20%. Raw materials (timber) are expected to remain in store for an average of 2 months before issue to production. Each unit of production is expected to be in process for 1 month. Finished goods will stay in the warehouse awaiting despatch to customers for about 3 months. Credit allowed by creditors is 2 months from date of delivery of raw materials. Credit given to debtors is 3 months from the despatch of goods. There is a regular production and sales cycle.

Discussion Point: The item wise estimate of working capital and the cost components in each thereof.

Forecast of Working Capital Requirements

	Period (months)	Total	Raw materials (Rs.)	Work in progress (Rs.)	Finished Goods (Rs.)	Debtors (Rs.)	Creditors (Rs.)
1 Materials							
(a) in stock	2		30.000				
(b) in work progress	1			15.000			
(c) in finished goods	3				45.000		
(d) credit to debtors	3					45.000	
							30.000
Total		1.35.000					
2 Wages							
(a) in work in progress	½			1.250*			
(b) in finished goods	3				7.500		
(c) credit to debtors	3					7.500	
Total		16.250					
3 Overheads							
(a) in work in progress	½			2.500*			
(b) in finished goods	3				15.000		
(c) Credit to debtors	3					15.000	
Total		32.500					
4 Profit							
(a) Credit to debtors	3					7.500	
Total		7.500					
Grand Total		1.91.250	30.000	18.750	67.500	75.000	30.000
Less : Creditors		30.000					
NET W.C.		1.61.250					

* To be discussed further

EXERCISE -2 ON WORKING CAPITAL – X CO-OPERATIVE LTD.

X Co-operative Ltd.. has been formed to procure oil seeds from its farmer members. process the seed into edible oil and meal and to market these products. It has set up a processing plant of capacity 1000 tons of oil seed processing per month. Harvest at the farmers' fields is due to commence shortly and the oil seeds are expected to arrive at the plant from next month i.e. November onwards and continue arriving till January. Estimates of oil seed procurement and sales of oil and meal and the various costs have been made. Production manager feels that it would be most appropriate to have level production through out at full capacity level.

It has now been decided to calculate the working capital requirements in detail for a five month period from November to March so that necessary arrangements can be made to raise the required funds before operations commence.

	Estimates
Arrival of oil seeds :	
November	1000 tons
December	2000 tons
January	2000 tons
Total :	5000 tons
Recovery :	
Oil (% of oilseeds)	15%
Meal	80%
Processing loss	5%

Expected sales - 100 tons of edible oil/ month and 600 tons of meal / month commencing from November itself.

Prices and costs :

Oil seed price Rs. 270/ton
Selling price :
Oil - Rs. 800/ton
Meal - Rs. 300/ton

Cost of fuel : Rs. 24/ ton of oil seed processed.
Labour : Rs. 1200/ month
Administrative and marketing costs. Rs. 2000/ month

Payment terms : Oilseed procurement on cash payment. Sales of oil and meal on one month's credit.

Discussion Points: The working capital requirement and the quantum of external funds needed to meet the requirement.

FINANCIAL CRITERIA FOR CAPITAL INVESTMENT DECISION

Introduction

An organization undertakes capital investment like investments in land. building. plant. machinery and vehicles principally on the premise that such investments will bring long-term benefits generally spanning over several years or even decades. Capital investments involve very critical decisions for several reasons.

- They have long-term consequences.
- Capital investment decisions are not easily reversible because they generally do not involve readily saleable commodities and often the capital equipment are tailor-made to meet an organisation's specific requirements.
- Capital investments involve substantial outlays.

The financial decision on a Capital Investment proposal would hinge on:

- Defining the relevant stream of costs and benefits associated with the investment
- Evaluating the cost-benefit stream for determining whether the investment would be worthwhile or not.

Project Evaluation: Alternate Methods

The following are the different financial methods of evaluating projects involving Capital Investment.

(a) Payback

(b) Accounting Rate of Return (ARR)

(c) Discounted Cash Flow Techniques

(i) Net Present Value (NPV)
(ii) Internal Rate of Return (IRR)

(a) Payback

Under this method the criteria of financial evaluation is the time required to payback the original investment. In other words. a project is accepted if the cash generated from the project equals the investment within a maximum stipulated time-frame. To illustrate. suppose a project involving an investment of Rs.1.00.000 generates the following cash flows:

Year	Cash Flows
1	Rs. 20.000
2	Rs. 20.000
3	Rs. 25.000
4	Rs. 35.000
5	Rs. 40.000

As can be seen from the above. the cash generations cumulate to Rs. 1.00.000 - the investment. in four years. That is. the project is said to have a payback of four years. If the criteria for acceptability is that the payback should not exceed four years or should not exceed five years. then this project would be acceptable.

The merits of this method are that it is based on actual cash flows and that the computations are simple and easily comprehendable. The main drawbacks of this method are:

(i) It ignores the time value of money.

(ii) In the choice of projects. a project with a lower payback period is preferred to that with a longer period. In other words. cash flows after the payback period are ignored. Thus a project with a longer payback but with better cash flows subsequently and hence better returns would be dropped and the project with a lower payback but lesser returns may be preferred.

b) Accounting Rate of Return (ARR or ROI)

This method. also known as the Return on Investment Method uses the accounting profit in relation to the investment. as the financial evaluation methodology. To illustrate. suppose the profits - either projected or actuals of a project involving an investment of Rs.1.00.000 are as follows:

Year	Profits
1	Rs. 10.000
2	Rs. 10.000
3	Rs. 15.000
4	Rs. 20.000
5	Rs. 25.000

The Accounting Rate of Return or ROI would be given by

$$\text{ROI} = \frac{\text{Average Profits}}{\text{Investment}}$$

$$ROI = \frac{1/5\ (Rs.\ 10,000 + Rs.\ 10,000 + Rs.\ 15,000 + Rs.\ 20,000 + Rs.\ 25,000)}{Rs.\ 1,00,000}$$

$$ROI = \frac{Rs.\ 16,000}{Rs.\ 1,00,000} = 16\%$$

A project is considered acceptable if the ROI is greater than or equal to a cut-off rate.

The merit of this method is its simplicity. The drawbacks are:

(i) This method is based on accounting profits – not cash flows. Two projects, exactly same in all respects may show different accounting profits if the accounting policies followed are different. Further, over the life of a project – cash flows is what matters for project selection – not book figures.

(ii) This method ignores the time value of money

(iii) There is no single accepted formula for ARR or ROI. Some calculate ARR before tax, some after, some before interest, some after and so on. ARR based on different formulae are bound to differ.

c) Discounted Cash Flow Techniques

From the drawbacks of the payback and ARR methods, one can infer that the most suitable method of project evaluation would be that which is based on:

- Cash flows
- Life of the project
- Time Value of Money

The discounted cash flow methods will be discussed in detail later. A guideline for drawing up the cash flows is given below:

Relevant Cash Flows

For applying the Discounted Cash Flow techniques, obviously the first step would be to identify the relevant cash flows, which would be subject to discounting. Costs would be the cash outflows and benefits the cash inflows, which would need to be drawn up, keeping the following principles in mind.

- **Cash Flow Principle:** Costs and benefits must be measured in terms of cash flows – costs are cash outflows and benefits cash inflows. Cash flows are emphasised because they represent the flow of purchasing power.

- **Post-tax Principle:** Cash flows must be measured in post-tax terms because they represent the net flow from the point of view of the firm. (It may be noted that cost of capital, which is the principal yardstick for appraising how far the cash flow stream is worthwhile, is also measured in post-tax terms for purposes of consistency).

- **Incremental Principle:** Cash flows must be measured in incremental terms. According to this principle, the changes in the cash flows of the firm arising from the adoption of the proposed project alone are relevant. In estimating incremental cash flows the following cautions should be observed.

(a) **Consider all incidental effects:** In addition to the direct cash flows of the project, all the incidental effects the project has on the rest of the organisation must be considered. It may enhance the profitability of some lines of existing activities because of complementary effects or it may detract from the profitability of some lines of existing activities because of competitiveness. All these effects should be considered.

(b) **Ignore sunk costs:** Sunk costs are bygones. Hence they do not matter for present decision-making.

(c) **Include opportunity costs:** If a project employs some resources available with the organisation it should be charged the opportunity costs of these resources. This should be done even if there are no explicit cash outflows arising from the use of these particular resources. For example, some spare capacity is used in the manufacture of a new product. The new product should be charged with the benefits that would arise from putting the spare capacity to its best alternative use. Of course, if there is no gainful alternative use of the spare capacity, its opportunity cost is nil.

- **Long-term Funds Principle:** In capital investment appraisal the principal focus is usually on the profitability of long-term funds. Hence cash flows relating to such funds need to be segregated.

- **Interest Exclusion Principle:** Interest on long-term debt should be excluded from the computation of profits and taxes. Why? The cost of capital used for appraising the cash flow stream reflects the time value of money. Hence interest cost, which represents the time cost of debt, must be excluded from the cash flow estimation. Otherwise we will let double counting take place.

To sum up, the costs and benefits of a project must be defined in terms of post-tax incremental cash flows relating to the long-term funds employed in the project.

Time-value of Money

The concept of Time-Value of Money is fundamental to finance and most financial decisions hinge on considerations of time value of money.

The principle of the time value of money is that a certain sum of money today is not worth the same as in the future and vice versa. A certain sum of money available today can be invested to earn interest or income (return) over the ensuing period. This gives rise to the concept of Future Value and Present Value as illustrated below:

Future Value of a Particular Amount of Money

If you invest Rs.1000/- today earning 10% annually compounded how much do you have after 1 year, 2 years, 3 years.

Table-1

Year	Beginning Amount (Rs.)	Interest for the year (Rs.)	Total interest (Rs.)	Ending Amount
1	1000	1000 x 0.1 = 100/-	100	1100 = 1000 x 1.1 $=1000\ (1+0.1)^1$
2	1100	1100 x 0.1 = 110/-	210	1210 = 1000 x 1.21 $= 1000\ (1+0.1)^2$
3	1210	1210 x 0.1 = 121/-	331	1331 = 1000 x 1.331 $= 1000\ (1+0.1)^3$

a) Thus, if we call the original or present amount as P_0 and the Ending amount or Future amount as FV, the interest or earning rate as i, then the amount after n years or future amount will be:

$FV_n = P_0\ (1+i)^n$, where FV_n is the future value after n years.

b) The factor $(1+i)^n$, is called the Future Value Interest factor which if multiplied to the original (present) amount will give the amount you will get at an interest or earning rate of i after n years.

Present Value of a Particular Amount of Money

Converse to the concept of future value, is the concept of present value.

From (a) $FV_n = P_0\ (1+i)^n$.

or $P_0 = \frac{FV_n}{(1+i)^n}$. Where P_0 is the Present Value of an amount of FV_n received after n years, at an interest or earning rate of i.

Thus, in the above illustration, the present value of Rs.1100 received after 1 year at 10% return rate is Rs.$\frac{1100}{(1+0.1)}$ = Rs.1000/-;

that of Rs. 1210 received after 2 years is Rs.$\frac{1210}{(1+0.1)^2}$ = Rs.1000/- and so on.

The factor $\frac{1}{(1+i)^n}$ is known as the Present Value Interest Factor or Discount Factor, which, if multiplied to the future amount, will give its present value.

The DCF Techniques

Considering the time value of money, it is clear that money at different points in time (as is obtainable in any project) is not directly comparable. For a proper comparison of the value of money at different points in time, it is clearly necessary to reduce all these time profiles to a single figure. To do this, two main techniques using the discounted cash flow method are used. They are the Net Present Value (NPV) and the Internal Rate of Return (IRR) methods.

Net Present Value

The NPV method consists of discounting all future cash flows to the present value by means of some appropriate rate of interest or earning. The rate of interest or earning to be used should reflect the minimum rate of return which is acceptable to the individual / organisation for a given investment. It works on the simple but fundamental principle that an investment is worth undertaking only if the present value of the cash inflows is at least equal to, if not greater than, the present value of the cash outflows arising from an investment. To put it another way, one should make investments in projects with a zero or positive net present value.

The computation is carried out as follows:

(i) Calculate the present value of each year's net cash flow by multiplying the projected cash flow by the appropriate discount factor as applicable to the minimum earning rate desired.

(ii) Add the computations to arrive at the single figure of net cash flows in present value.

If the result of (ii) (which is NPV) is zero or positive, i.e., the present values of the cash inflows is at least equal to, if not greater than, the present values of all cash outflows, the investment would be acceptable at the discount rate used: if the NPV is negative, the investment should be rejected.

The Internal Rate of Return

The internal rate of return is an alternative approach used in making investment decisions, which also take into account the time value of money. The IRR represents the return (in present terms) earned on an investment over its economic life. It is defined as that interest rate which, when applied to the cash flows generated by an investment, will equate the present value of the cash inflows to the present value of the cash outflows. In other words, it is the discount rate which will cause the NPV of an investment to be zero.

As can be seen thereof – though Future Value or Present Value are two different ways of looking at the same cash flows, it is the concept of Present Value and its extensions that are in more usage in financial decisions. The reasons for this would become apparent as we proceed further into exercises and discussions.

Now we will take up some simple and relevant exercises which will be followed by further extensions of these concepts.

Exercise 1 on Time Value of Money

A friend of yours, suggests that you deposit with him/her Rs. 1027/- today and he/she would give you back Rs. 1331/- after 3 years. He/she is fully credit worthy. Your decision on placing the deposit, or otherwise, would not be guided by any sentimental factor- but would be on financial reasoning.

a) Would you place the deposit if your expected earning rate is 10% per annum?
b) What will the decision be if your expected earning rate is 8% per annum?
c) What is the return rate he/she is implicitly offering?

Exercise 2 on Time Value of Money/Capital Budgeting

You are examining a Project which has the following Cash Flows.

Year	Cash Flow (Rs.)
0	(1200)
1	100
2	600
3	1083

Your minimum expected return is 16%. Evaluate the acceptability or otherwise of the project from both the NPV and IRR angles.

Exercise 3 on Capital Investment Decision

In exercise-2, we had evaluated the acceptability or otherwise of a Project from the NPV & IRR angles, given that the minimum expected return (or hurdle or discount rate) was 16%. The Cash Flows of the Project were as under:

Year	Cash Flow (Rs.)
0	(1200)
1	100
2	600
3	1083

So far in the exercises which we had done, the minimum expected return (hurdle or discount rate) was given. Now let us evaluate the above Project after determining the hurdle or discount rate given the following information:

The Project Cost of Rs. 1200/- is financed as follows:-

50% from the share holders' equity
50% from Long term loan at an interest of 10% per annum

The Share holders expect a return of 12% per annum for their investment. As is prevalent in most countries, interest on loan is a tax deductible expense but dividend is not. The tax rate is 40%.

Farmers' Co-operative Project*

X. A Producers' Co-operative has been set up in a city to procure and market. a wide range of Agri produce of the farmers in neighbouring areas. The procured produce would be brought to a central storage (CS) to be set up in the city and sold through 200 Retail Shops – both belonging to the Co-operative. The CS would be a very sophisticated facility consisting of sorting and grading lines and cold stores with temperature and humidity controls. The Co-operative has made estimates of project costs. likely activity levels and sales and revenues which are as below:

(i) Cost of CS and Associate facilities — Rs. 30 crores — to be fully incurred in the year before operations i.e. Year 0

(ii) Cost of Retail Shops — Rs. 6 crores — to be fully incurred in the year before operations i.e. Year 0

(iii) Projected activity levels

Year	1	2	3	4	5
Quantity of Agri-produce to be sold (tons)	80.000	1.00.000	1.10.000	1.21.000	1.21.000
Weighted average variable Cost/ton (Rs.) excluding interest	7.000	7.000	7.000	7.000	7.000
Weighted average selling price/ton (Rs.)	8.500	8.500	8.500	8.500	8.500
(iv) Fixed costs (Rs.) (other than interest and depreciation)	2 crores	2 crores	2 crores	2 crores	2 crores

(v) Working Capital Requirement : It has been found that the working capital requirement for each year consists of a minimum (permanent component) and a fluctuating part. Both the minimum component and the fluctuating part increase directly in proportion to the quantity being sold in the year. Management has decided to meet 50% of the minimum (permanent) component through retained earnings / equity infusions and 50% through medium term loan and the fluctuating component would be met through short term Bank Borrowings at an interest rate of 12% per annum. The month wise working capital computation for the first year are as follows:

* Imaginary Project – for classroom discussions formulated by Prof. S. Roy of the Institute of Rural Management. Anand.

April	– Rs. 3 crores	October	– Rs. 5 crores
May	– Rs. 3 crores	November	– Rs. 4 crores
June	– Rs. 4 crores	December	– Rs. 4 crores
July	– Rs. 5 crores	January	– Rs. 3 crores
August	– Rs. 6 crores	February	– Rs. 3 crores
September	– Rs. 5 crores	March	– Rs. 3 crores

(vi)	Depreciation for tax purposes	25% on written down value
(vii)	Tax rate	50%
(viii)	Salvage value of CS and retail shops	Rs. 9 crores (Ignore capital gains tax)
(ix)	Project financing	(i) <u>CS, Retail shops and permanent working capital</u> 50% through members' equity and 50% Medium term loan. The loan carries an interest of 12% per annum. The members expect a minimum return of 14% on equity. (ii) <u>Fluctuating Working Capital</u> Short term bank borrowings at an interest of 12% per annum.

(1) You are required to prepare the relevant project cash flows as would be necessary to evaluate the economics of the project on NPV and IRR basis.

(2) Evaluate the project from the financial angle.

Assign

Table I

Future value interest factor of $1 at i % at the end of n periods ($FVIF_{i,n}$)

$(FVIF_{i,n}) = (1 + i)^n$

PERIOD (n)	INTEREST RATE (i)												PERIOD (n)
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	
1	1.010	1.020	1.030	1.040	1.050	1.060	1.070	1.080	1.090	1.100	1.100	1.120	1
2	1.020	1.040	1.061	1.082	1.102	1.124	1.145	1.166	1.188	1.210	1.232	1.254	2
3	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	1.331	1.368	1.405	3
4	1.041	1.082	1.126	1.170	1.216	1.262	1.311	1.360	1.412	1.464	1.518	1.574	4
5	1.051	1.104	1.159	1.217	1.276	1.338	1.403	1.469	1.539	1.611	1.685	1.762	5
6	1.062	1.126	1.194	1.265	1.340	1.419	1.501	1.587	1.677	1.772	1.870	1.974	6
7	1.072	1.149	1.230	1.316	1.407	1.504	1.606	1.714	1.828	1.949	2.076	2.211	7
8	1.083	1.172	1.267	1.369	1.477	1.594	1.718	1.851	1.993	2.144	2.305	2.476	8
9	1.094	1.195	1.305	1.423	1.551	1.689	1.838	1.999	2.172	2.358	2.558	2.773	9
10	1.105	1.219	1.344	1.480	1.629	1.791	1.967	2.159	2.367	2.594	2.839	3.106	10
11	1.116	1.243	1.384	1.539	1.710	1.898	2.105	2.332	2.580	2.853	3.152	3.479	11
12	1.127	1.268	1.426	1.601	1.796	2.012	2.252	2.518	2.813	3.138	3.498	3.896	12
13	1.138	1.294	1.469	1.665	1.886	2.133	2.410	2.720	3.066	3.452	3.883	4.363	13
14	1.149	1.319	1.513	1.732	1.980	2.261	2.579	2.937	3.342	3.797	4.310	4.887	14
15	1.161	1.346	1.558	1.801	2.079	2.397	2.759	3.172	3.642	4.177	4.785	5.474	15
16	1.173	1.373	1.605	1.873	2.183	2.540	2.952	3.426	3.970	4.595	5.311	6.130	16
17	1.184	1.400	1.653	1.948	2.292	2.693	3.159	3.700	4.328	5.054	5.895	6.866	17
18	1.196	1.428	1.702	2.026	2.407	2.854	3.380	3.996	4.717	5.560	6.544	7.690	18
19	1.208	1.457	1.754	2.107	2.527	3.026	3.617	4.316	5.142	6.116	7.263	8.613	19
20	1.220	1.486	1.806	2.191	2.653	3.207	3.870	4.661	5.604	6.727	8.062	9.646	20
25	1.282	1.641	2.094	2.666	3.386	4.292	5.427	6.848	8.623	10.835	13.585	17.000	25
30	1.348	1.811	2.427	3.243	4.322	5.743	7.612	10.063	13.268	17.449	22.892	29.960	30
35	1.417	2.000	2.814	3.946	5.516	7.686	10.677	14.785	20.414	28.102	38.575	52.800	35
40	1.489	2.208	3.262	4.801	7.040	10.286	14.974	21.725	31.409	45.259	65.001	93.051	40
50	1.645	2.692	4.384	7.107	11.467	18.420	29.457	46.902	74.358	117.391	184.565	289.002	50

Table I (Continued)
Future value interest factor of $1 at i % at the end of n periods ($FVIF_{i,n}$)
$(FVIF_{i,n}) = (1 + i)^n$

PERIOD (n)	INTEREST RATE (i)												PERIOD (n)
	13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	40%	50%	
1	1.130	1.140	1.150	1.160	1.170	1.180	1.190	1.200	1.250	1.300	1.400	1.500	1
2	1.277	1.300	1.322	1.346	1.369	1.392	1.416	1.440	1.563	1.690	1.960	2.250	2
3	1.443	1.482	1.521	1.561	1.602	1.643	1.685	1.728	1.953	2.197	2.744	3.375	3
4	1.630	1.689	1.749	1.811	1.874	1.939	2.005	2.074	2.441	2.856	3.842	5.063	4
5	1.842	1.925	2.011	2.100	2.192	2.288	2.386	2.488	3.052	3.713	5.378	7.594	5
6	2.082	2.195	2.313	2.436	2.565	2.700	2.840	2.986	3.815	4.827	7.530	11.391	6
7	2.353	2.502	2.660	2.826	3.001	3.185	3.379	3.583	4.768	6.275	10.541	17.086	7
8	2.658	2.853	3.059	3.278	3.511	3.759	4.021	4.300	5.960	8.157	14.758	25.629	8
9	3.004	3.252	3.518	3.803	4.108	4.435	4.785	5.160	7.451	10.604	20.661	38.443	9
10	3.395	3.707	4.046	4.411	4.807	5.234	5.696	6.192	9.313	13.786	28.925	57.665	10
11	3.836	4.226	4.652	5.117	5.624	6.176	6.777	7.430	11.642	17.922	40.496	86.498	11
12	4.335	4.818	5.350	5.936	6.580	7.288	8.064	8.916	14.552	23.298	56.694	129.746	12
13	4.898	5.492	6.153	6.886	7.699	8.599	9.596	10.699	18.190	30.288	79.372	194.620	13
14	5.535	6.261	7.076	7.988	9.007	10.147	11.420	12.839	22.737	39.374	111.120	291.929	14
15	6.254	7.138	8.137	9.266	10.539	11.974	13.590	15.407	28.422	51.186	155.568	437.894	15
16	7.067	8.137	9.358	10.748	12.330	14.129	16.172	18.488	35.527	66.542	217.795	656.841	16
17	7.986	9.276	10.761	12.468	14.426	16.672	19.244	22.186	44.409	86.504	304.914	985.261	17
18	9.024	10.575	12.375	14.463	16.879	19.673	22.901	26.623	55.511	112.455	426.879	1477.892	18
19	10.197	12.056	14.232	16.777	19.748	23.214	27.252	31.948	69.389	146.192	597.630	2216.838	19
20	11.523	13.743	16.367	19.461	23.106	27.393	32.429	38.338	86.736	190.050	836.683	3325.257	20
25	21.231	26.462	32.919	40.874	50.658	62.669	77.388	95.396	264.698	705.641	4499.880	25251.168	25
30	39.116	50.950	66.212	85.850	111.065	143.371	184.675	237.376	807.794	2620.000	24201.432	191751	30
35	72.069	98.100	133.176	180.314	243.503	327.997	440.701	590.668	2465.190	9727.860	130161	1456110	35
40	139.782	188.884	267.864	378.721	533.869	750.378	1051.668	1469.772	7523.164	36118.865	700038	11057332	40
50	450.736	700.233	1083.657	1670.704	2566.215	3927.357	5988.914	9100.438	70064.923	497929.223	20248916	637621500	50

Table II
Present value interest factor of $1 at i % for n periods ($PVIF_{i,n}$)
$(PVIF_{i,n}) = 1 / (1 + i)^n$

PERIOD (n)	INTEREST RATE (i)												PERIOD (n)
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	
1	.990	.980	.971	.962	.952	.943	.935	.926	.917	.909	.901	.893	1
2	.980	.961	.943	.925	.907	.890	.873	.857	.842	.826	.812	.797	2
3	.971	.942	.915	.889	.864	.840	.816	.794	.772	.751	.731	.712	3
4	.961	.924	.888	.855	.823	.792	.763	.735	.708	.683	.659	.636	4
5	.951	.906	.863	.822	.784	.747	.713	.681	.650	.621	.593	.567	5
6	.942	.888	.837	.790	.746	.705	.666	.630	.596	.564	.535	.507	6
7	.933	.871	.813	.760	.711	.665	.623	.583	.547	.513	.482	.452	7
8	.923	.853	.789	.731	.677	.627	.582	.540	.502	.467	.434	.404	8
9	.914	.837	.766	.703	.645	.592	.544	.500	.460	.424	.391	.361	9
10	.905	.820	.744	.676	.614	.558	.508	.463	.422	.386	.352	.322	10
11	.896	.804	.722	.650	.585	.527	.475	.429	.388	.350	.317	.287	11
12	.887	.789	.701	.625	.557	.497	.444	.397	.356	.319	.286	.257	12
13	.879	.773	.681	.601	.530	.469	.415	.368	.326	.290	.258	.229	13
14	.870	.758	.661	.577	.505	.442	.388	.340	.299	.263	.232	.205	14
15	.861	.743	.642	.555	.481	.417	.362	.315	.275	.239	.209	.183	15
16	.853	.728	.623	.534	.458	.394	.339	.292	.252	.218	.188	.163	16
17	.844	.714	.605	.513	.436	.371	.317	.270	.231	.198	.170	.146	17
18	.836	.700	.587	.494	.416	.350	.296	.250	.212	.180	.153	.130	18
19	.828	.686	.570	.475	.396	.331	.277	.232	.194	.164	.138	.116	19
20	.820	.673	.554	.456	.377	.312	.258	.215	.178	.149	.124	.104	20
25	.780	.610	.478	.375	.295	.233	.184	.146	.116	.092	.074	.059	25
30	.742	.552	.412	.308	.231	.174	.131	.099	.075	.057	.044	.033	30
35	.706	.500	.355	.253	.181	.130	.094	.068	.049	.036	.026	.019	35
40	.672	.453	.307	.208	.142	.097	.067	.046	.032	.022	.015	.011	40
50	.608	.372	.228	.141	.087	.054	.034	.021	.013	.009	.005	.003	50

Table II (Continued)
Present value interest factor of $1 at i % for n periods ($PVIF_{i,n}$)
$(PVIF_{i,n}) = 1 / (1 + i)^n$

PERIOD (n)	INTEREST RATE (i)												PERIOD (n)
	13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	40%	50%	
1	.885	.877	.870	.862	.855	.847	.840	.833	.800	.769	.714	.667	1
2	.783	.769	.756	.743	.731	.718	.706	.694	.640	.592	.510	.444	2
3	.693	.675	.658	.641	.624	.609	.593	.579	.512	.455	.364	.296	3
4	.613	.592	.572	.552	.534	.516	.499	.482	.410	.350	.260	.198	4
5	.543	.519	.497	.476	.456	.437	.419	.402	.328	.269	.186	.132	5
6	.480	.456	.432	.410	.390	.370	.352	.335	.262	.207	.133	.088	6
7	.425	.400	.376	.354	.333	.314	.296	.279	.210	.159	.095	.059	7
8	.376	.351	.327	.305	.285	.266	.249	.233	.168	.123	.068	.039	8
9	.333	.308	.284	.263	.243	.225	.209	.194	.134	.094	.048	.026	9
10	.295	.270	.247	.227	.208	.191	.176	.162	.107	.073	.035	.017	10
11	.261	.237	.215	.195	.178	.162	.148	.135	.086	.056	.025	.012	11
12	.231	.208	.187	.168	.152	.137	.124	.112	.069	.043	.018	.008	12
13	.204	.182	.163	.145	.130	.116	.104	.093	.055	.033	.013	.005	13
14	.181	.160	.141	.125	.111	.099	.088	.078	.044	.025	.009	.003	14
15	.160	.140	.123	.108	.095	.084	.074	.065	.035	.020	.006	.002	15
16	.141	.123	.107	.093	.081	.071	.062	.054	.028	.015	.005	.002	16
17	.125	.108	.093	.080	.069	.060	.052	.045	.023	.012	.003	.001	17
18	.111	.095	.081	.069	.059	.051	.044	.038	.018	.009	.002	.001	18
19	.098	.083	.070	.060	.051	.043	.037	.031	.014	.007	.002	.000	19
20	.087	.073	.061	.051	.043	.037	.031	.026	.012	.005	.001	.000	20
25	.047	.038	.030	.024	.020	.016	.013	.010	.004	.001	.000	.000	25
30	.026	.020	.015	.012	.009	.007	.005	.004	.001	.000	.000	.000	30
35	.014	.010	.008	.006	.004	.003	.002	.002	.000	.000	.000	.000	35
40	.008	.005	.004	.003	.002	.001	.001	.001	.000	.000	.000	.000	40
50	.002	.001	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	50

Table III

Future value interest factor of an (ordinary) annuity of $1 per period at i % for n periods ($FVIFA_{i,n}$)

$$\left(FVIFA_{i,n}\right) = \sum_{t=1}^{n}(1+i)^{n-t} = \frac{(1+i)^{n}-1}{i}$$

PERIOD (n)	INTEREST RATE (i)												PERIOD (n)
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1
2	2.010	2.020	2.030	2.040	2.050	2.060	2.070	2.080	2.090	2.100	2.110	2.120	2
3	3.030	3.060	3.091	3.122	3.153	3.184	3.215	3.246	3.278	3.310	3.342	3.374	3
4	4.060	4.122	4.184	4.246	4.310	4.375	4.440	4.506	4.573	4.641	4.710	4.779	4
5	5.101	5.204	5.309	5.416	5.526	5.637	5.751	5.867	5.985	6.105	6.228	6.353	5
6	6.152	6.308	6.468	6.633	6.802	6.975	7.153	7.336	7.523	7.716	7.913	8.115	6
7	7.214	7.434	7.662	7.898	8.142	8.394	8.654	8.923	9.200	9.487	9.783	10.089	7
8	8.286	8.583	8.892	9.214	9.549	9.897	10.260	10.637	11.028	11.436	11.859	12.300	8
9	9.369	9.755	10.159	10.583	11.027	11.491	11.978	12.488	13.021	13.579	14.164	14.776	9
10	10.462	10.950	11.464	12.006	12.578	13.181	13.816	14.487	15.193	15.937	16.722	17.549	10
11	11.567	12.169	12.803	13.486	14.207	14.972	15.784	16.645	17.560	18.531	19.561	20.655	11
12	12.683	13.412	14.192	15.026	15.917	16.870	17.888	18.977	20.141	21.384	22.713	24.133	12
13	13.809	14.680	15.618	16.627	17.713	18.882	20.141	21.495	22.953	24.523	26.212	28.029	13
14	14.947	15.974	17.086	18.292	19.599	21.015	22.550	24.215	26.019	27.975	30.095	32.393	14
15	16.097	17.293	18.599	20.024	21.579	23.276	25.129	27.152	29.361	31.772	34.405	37.280	15
16	17.258	18.639	20.157	21.825	23.657	25.673	27.888	30.324	33.003	35.950	39.190	42.753	16
17	18.430	20.012	21.762	23.698	25.840	28.213	30.840	33.750	36.974	40.545	44.501	48.884	17
18	19.615	21.412	23.414	25.645	28.132	30.906	33.999	37.450	41.301	45.599	50.396	55.750	18
19	20.811	22.841	25.117	27.671	30.539	33.760	37.379	41.446	46.018	51.159	56.939	63.440	19
20	22.019	24.297	26.870	29.778	33.066	36.786	40.995	45.762	51.160	57.275	64.203	72.052	20
25	28.243	32.030	36.459	41.646	47.727	54.865	63.249	73.106	84.701	98.347	114.413	133.334	25
30	34.785	40.568	47.575	56.085	66.439	79.058	94.461	113.283	136.308	164.494	199.021	241.333	30
35	41.660	49.994	60.462	73.652	90.320	111.435	138.237	172.317	215.711	271.024	341.590	431.663	35
40	48.886	60.402	75.401	95.026	120.800	154.762	199.635	259.057	337.882	442.593	581.826	767.091	40
50	64.463	84.579	112.797	152.667	209.348	290.336	406.529	573.770	815.084	1163.909	1668.771	2400.018	50

Table III (continued)
Future value interest factor of an (ordinary) annuity of $1 per period at i % for n periods ($FVIFA_{i,n}$)

$$\left(FVIFA_{i,n}\right) = \sum_{t=1}^{n}(1+i)^{n-t} = \frac{(1+i)^{n}-1}{i}$$

PERIOD (n)	INTEREST RATE (i)												PERIOD (n)
	13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	40%	50%	
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1
2	2.130	2.140	2.150	2.160	2.170	2.180	2.190	2.200	2.250	2.300	2.400	2.500	2
3	3.407	3.440	3.473	3.506	3.539	3.572	3.606	3.640	3.813	3.990	4.360	4.750	3
4	4.850	4.921	4.993	5.066	5.141	5.215	5.291	5.368	5.766	6.187	7.104	8.125	4
5	6.480	6.610	6.742	6.877	7.014	7.154	7.297	7.442	8.207	9.043	10.946	13.188	5
6	8.323	8.536	8.754	8.977	9.207	9.442	9.683	9.930	11.259	12.756	16.324	20.781	6
7	10.405	10.730	11.067	11.414	11.772	12.142	12.523	12.916	15.073	17.583	23.853	32.172	7
8	12.757	13.233	13.727	14.240	14.773	15.327	15.902	16.499	19.842	23.858	34.395	49.258	8
9	15.416	16.085	16.786	17.519	18.285	19.086	19.923	20.799	25.802	32.015	49.153	74.887	9
10	18.420	19.337	20.304	21.321	22.393	23.521	24.709	25.959	33.253	42.619	69.814	113.330	10
11	21.814	23.045	24.349	25.733	27.200	28.755	30.404	32.150	42.566	56.405	98.739	170.995	11
12	25.650	27.271	29.002	30.850	32.824	34.931	37.180	39.581	54.208	74.327	139.235	257.493	12
13	29.985	32.089	34.352	36.786	39.404	42.219	45.244	48.497	68.760	97.625	195.929	387.239	13
14	34.883	37.581	40.505	43.672	47.103	50.818	54.841	59.196	86.949	127.913	275.300	581.859	14
15	40.417	43.842	47.580	51.660	56.110	60.965	66.261	72.035	109.687	167.286	386.420	873.788	15
16	46.672	50.980	55.717	60.925	66.649	72.939	79.850	87.442	138.109	218.472	541.988	1311.682	16
17	53.739	59.118	65.075	71.673	78.979	87.068	96.022	105.931	173.636	285.014	759.784	1968.523	17
18	61.725	68.394	75.836	84.141	93.406	103.740	115.266	128.117	218.045	371.518	1064.697	2953.784	18
19	70.749	78.969	88.212	98.603	110.285	123.414	138.166	154.740	273.556	483.973	1491.576	4431.676	19
20	80.947	91.025	102.444	115.380	130.033	146.638	165.418	186.688	342.945	630.165	2089.206	6648.513	20
25	155.620	181.871	212.793	249.214	292.105	342.603	402.042	471.981	1054.791	2348.803	11247.199	50500	25
30	293.199	356.787	434.745	530.312	647.439	790.948	966.712	1181.882	3227.174	8729.985	60501	383500	30
35	546.681	693.573	881.170	1120.713	1426.491	1120.713	2314.214	2948.341	9856.761	32423	325400	2912217	35
40	1013.704	1342.025	1779.090	2360.757	3134.522	4163.21	5529.829	7343.858	30089	120393	1750092	22114663	40
50	3459.507	4994.521	7217.716	10435.649	15089.502	21813.1	31515	45497	280256	1659761	50622288	1275242998	50

Table IV

Present value interest factor of an (ordinary) annuity of $1 per period at i % for n periods ($PVIFA_{i,n}$)

$$\left(PVIFA_{i,n}\right) = \sum_{t=1}^{n} 1 \Big/ (1+i)^{t} = \frac{1-[1/(1+i)^{n}]}{i}$$

PERIOD (n)	INTEREST RATE (i)												PERIOD (n)
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	0.901	0.893	1
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736	1.713	1.690	2
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487	2.444	2.402	3
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170	3.102	3.037	4
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791	3.696	3.605	5
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355	4.231	4.111	6
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868	4.712	4.564	7
8	7.652	7.326	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335	5.146	4.968	8
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759	5.537	5.328	9
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145	5.889	5.650	10
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495	6.207	5.938	11
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814	6.492	6.194	12
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103	6.750	6.424	13
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367	6.982	6.628	14
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.560	8.061	7.606	7.191	6.811	15
16	14.718	13.578	12.561	11.652	10.838	10.106	9.447	8.851	8.313	7.824	7.379	6.974	16
17	15.562	14.292	13.166	12.166	11.274	10.477	9.763	9.122	8.544	8.022	7.549	7.120	17
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.372	8.756	8.201	7.702	7.250	18
19	17.226	15.679	14.324	13.134	12.085	11.158	10.336	9.604	8.950	8.365	7.839	7.366	19
20	18.046	16.352	14.877	13.590	12.462	11.470	10.594	9.818	9.129	8.514	7.963	7.469	20
25	22.023	19.524	17.413	15.622	14.094	12.784	11.654	10.675	9.823	9.077	8.422	7.843	25
30	25.808	22.396	19.601	17.292	15.373	13.765	12.409	11.258	10.274	9.427	8.694	8.055	30
35	29.409	24.999	21.487	18.665	16.374	14.498	12.948	11.655	10.567	9.644	8.855	8.176	35
40	32.835	27.356	23.115	19.793	17.159	15.046	13.332	11.925	10.757	9.779	8.951	8.244	40
50	39.196	31.424	25.730	21.482	18.256	15.762	13.801	12.233	10.962	9.915	9.042	8.304	50

Table IV (continued)

Present value interest factor of an (ordinary) annuity of $1 per period at i % for n periods ($PVIFA_{i,n}$)

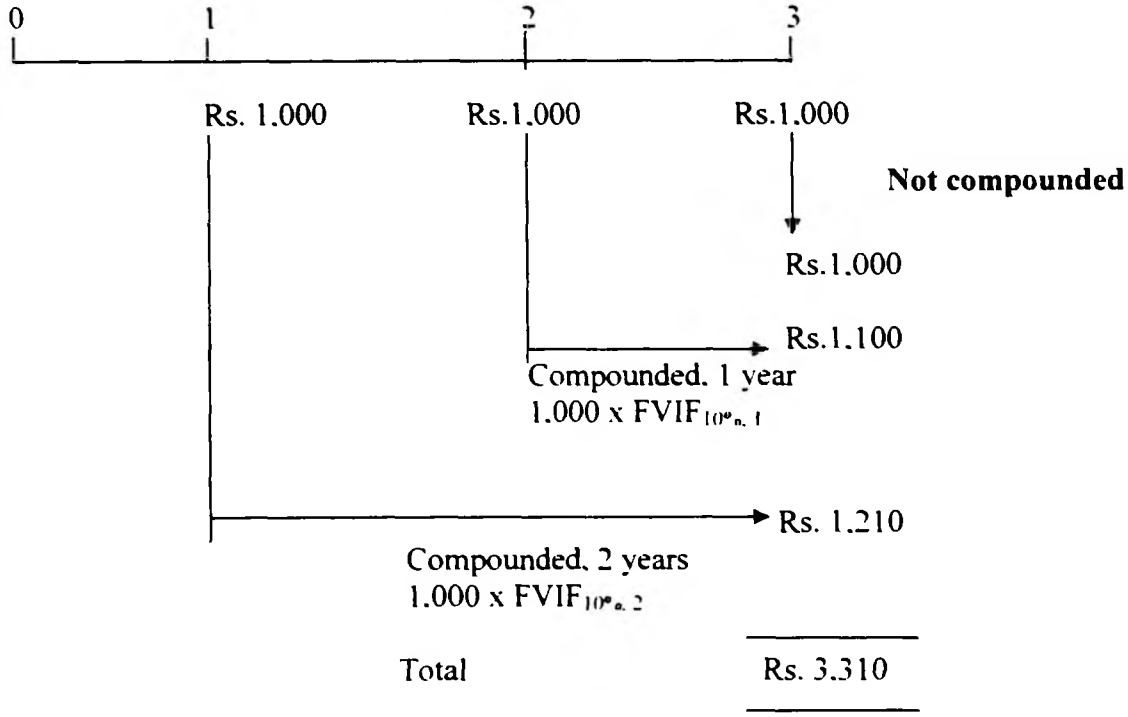
$$(PVIFA_{i,n}) = \sum_{t=1}^{n} 1/(1+i)^t = \frac{1-[1/(1+i)^n]}{i}$$

PERIOD (n)	INTEREST RATE (i)												PERIOD (n)
	13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	40%	50%	
1	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	0.800	0.769	0.714	0.667	1
2	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528	1.440	1.361	1.224	1.111	2
3	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106	1.952	1.816	1.589	1.407	3
4	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589	2.362	2.166	1.849	1.605	4
5	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991	2.689	2.436	2.035	1.737	5
6	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326	2.951	2.643	2.168	1.824	6
7	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605	3.161	2.802	2.263	1.883	7
8	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837	3.329	2.925	2.331	1.922	8
9	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031	3.463	3.019	2.379	1.948	9
10	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192	3.571	3.092	2.414	1.965	10
11	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327	3.656	3.147	2.438	1.977	11
12	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439	3.725	3.190	2.456	1.985	12
13	6.122	5.842	5.583	5.342	5.118	4.910	4.715	4.533	3.780	3.223	2.469	1.990	13
14	6.302	6.002	5.724	5.468	5.229	5.008	4.802	4.611	3.824	3.249	2.478	1.993	14
15	6.462	6.142	5.847	5.575	5.324	5.092	4.876	4.675	3.859	3.268	2.484	1.995	15
16	6.604	6.265	5.954	5.668	5.405	5.162	4.938	4.730	3.887	3.283	2.489	1.997	16
17	6.729	6.373	6.047	5.749	5.475	5.222	4.990	4.775	3.910	3.295	2.492	1.998	17
18	6.840	6.467	6.128	5.818	5.534	5.273	5.033	4.812	3.928	3.304	2.494	1.999	18
19	6.938	6.550	6.198	5.877	5.584	5.316	5.070	4.843	3.942	3.311	2.496	1.999	19
20	7.025	6.623	6.259	5.929	5.628	5.353	5.101	4.870	3.954	3.316	2.497	1.999	20
25	7.330	6.873	6.464	6.097	5.766	5.467	5.195	4.948	3.985	3.329	2.499	2.000	25
30	7.496	7.003	6.566	6.177	5.829	5.517	5.235	4.979	3.995	3.332	2.500	2.000	30
35	7.586	7.070	6.617	6.215	5.858	5.539	5.251	4.992	3.998	3.333	2.500	2.000	35
40	7.634	7.105	6.642	6.233	5.871	5.548	5.258	4.997	3.999	3.333	2.500	2.000	40
50	7.675	7.133	6.661	6.246	5.880	5.554	5.262	4.999	4.000	3.333	2.500	2.000	50

THE CONCEPT OF ANNUITY

Now, that we are familiar with the concept of Time Value of Money and its applications to our work situations, we can extend the principles to a phenomenon of great practical utility which is, the concept of Annuity. An annuity represents a series of equal payments or receipts over a specified number of periods.

To illustrate this principle, let us assume that a savings scheme requires us to deposit Rs.1.000/- at the end of each year for three years and the scheme pays an interest of 10% per annum, compounded annually. To calculate the amount of that we would receive on maturity of the scheme, we can take recourse to our concept of time value of money, which is shown below:



Expressed mathematically, the future value (FV) of the annuity is

$$FV = 1.000\{(1 + 0.10)^2 + (1 + 0.10)^1 + (1 + 0.10)^0\}$$

To generalize, if 'R' represents the periodic equal payments, 'n' the number of periods and 'i' the interest rate, the Future Value (FV) of an annuity is given by:

$$FV = R\{(1 + i)^{n-1} + (1 + i)^{n-2} + \ldots . (1 + i)^0\}$$

As can be seen, the terms within the brackets, which are basically the interest factors, constitute terms in Geometric Progression whose sum is :

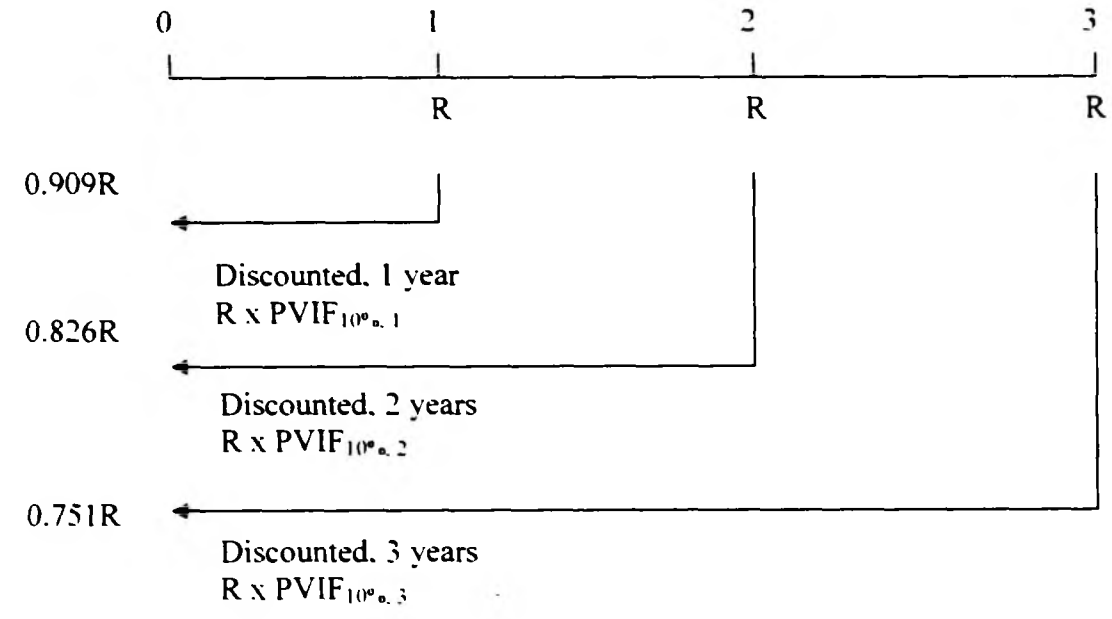
$[\{(1+i)^n - 1\}/i]$

Hence, the future value of the annuity,

$FV = R [\{(1+i)^n - 1\}/i]$

Converse to the Concept of Future Value of an annuity, is the Concept of Preset Value of an annuity. To illustrate, let us assume that a loan of Rs. 2,486 has been given to a beneficiary at an interest rate of 10% per annum and the beneficiary has to repay in 3 equal annual instalments, say 'R'. We want to determine the equal annual instalment amount i.e., 'R'.

The amount of R should be such that the sum of their present values, discounted at 10% per annum should equal the loan amount of Rs. 2486/-.



2.486R	Total

In this case, 2.486R = Rs.2,486

Hence, R = Rs.2,486 = Rs.1,000/-
2.486

This, each annual instalment has to be Rs.1,000/-.

Fortunately, just as in the case of Future Value Interest Factor of an annuity, the Present Value Interest Factor is also a sum of a series in Geometric Progression and it comes to

$\left[\left\{1-\left(\frac{1}{1+i}\right)^{n}\right\} /i\right]$ Hence, present Value of an annuity (PV) is given by

$$\mathbf{PV} = \mathbf{R}\left[\left\{1-\left(\frac{1}{1+i}\right)^{n}\right\} /i\right]$$

Where, 'i' is the interest rate, 'n' the number of periods and 'R' the equal instalment.

Exercise 1 on Annuity

You wish to build up a savings of Rs. 1 lakh by the end of 5 years. How much equal amount should you save annually if your savings earn an interest of 12% compounded annually. Please work out initially assuming instalments being placed at the end of each period. Thereafter, work out based on instalments placed at the beginning of each period.

Exercise 2 on Annuity

A monthly Recurring Deposit Scheme of 5 – years tenure gives a nominal interest rate of 8% per annum. applied monthly. You wish to save an equal amount each month such that at the termination of the scheme. the amount accumulates to Rs.1.00.000. What should be your equal monthly deposit. assuming (a) deposits at the end of each month (b) deposits at the beginning of each month?

Exercise-3 on Annuity

1. An NGO has received a credit of Rs.10 million from a National Development Finance Body. The funds are to be used for financing to needy, deserving people to enable them to engage in some self-sustaining ventures. The credit carries a nominal annual interest rate of 10% and has to be repaid by the NGO in 5 equal half yearly instalments. The NGO in turn, would finance Rs. 20,000/- to each of 500 beneficiaries who have already been identified. The beneficiaries would also have to repay the amount to the NGO in 5 equal half yearly instalments but they would be charged a nominal annual interest of 12% per annum.

   Discussion Points:

   a) What would be the half yearly instalment that the NGO has to pay to the National Development Finance Body; and.

   b) What would be the half yearly instalment that each beneficiary has to pay to the NGO.

   c) Prepare the loan amortization schedule for the loan taken and repaid by the NGO.

Exercise-4 on Annuity

In exercise 3 on annuity, it was implicitly assumed that the recovery from the beneficiaries would be 100% that is, all would repay the loan. In real life, some defaults cannot be ruled out. Also, there would be some administrative or managerial costs. Let us assume that the NGO incurs costs of Rs. 20,000/- per half-year towards managing of the loan scheme. It needs to know, the amount or percentage of default that it can sustain.

Exercise 5 on Annuity/EMI

Attached is a copy of an advertisement of a large Housing Finance Company.

Assuming that we had to calculate the EMI for the 6 year term per Rs. 1.00.000 at the rate of interest stated by them i.e.. 10% per annum. How would we do it? Does it come to the same figure indicated by them i.e.. Rs. 1853/-?

Additional Note on Annuity*

Annuity Revisited

One may recall from the earlier discussion that annuity represents a series of equal payments or receipts over a specified number of periods. The future value interest factor for an ordinary annuity is given by the expression, $FV = \{(1+i)^n - 1\}/i$, and the present value interest factor is given by the expression,

$$PV = \left\{1 - \frac{1}{(1+i)^n}\right\} \Big/ i \text{ where,}$$

'i' is the interest rate during each period and 'n' is the number of periods.

The formula for the future value interest factor can be used to determine the equal periodic deposits to be made to build up a certain amount of savings at the end of a specified number of periods, at a given interest rate. For instance, if there is a 60 month recurring deposit scheme, paying an interest of 8 percent per annum, compounded monthly, then the amount of deposit, say 'R', to be made at the end of each month so as to build up Rs.1,00,000 by maturity, can be found out as follows:

$$R\left\{\left(1+\frac{0.08}{12}\right)^{60} - 1\right\}(0.08)/_{12} = \text{Rs. } 1,00,000$$

Or R x 73.4769 = Rs. 1,00,000

R = Rs.1,00,000 / 73.4769 = Rs.1361/-

The formula for the Present Value Interest factor can be used to determine the equal periodic loan repayment instalments for a given amount of loan at a given interest rate bearing a given repayment period. Thus, if a housing loan of Rs.1,00,000 carrying an interest of 10% per annum is to be repaid in 72 equal monthly instalment made at the end of each month then the EMI, say 'R', can be found out as follows:

$$R\left\{1 - \left(\frac{1}{1+\frac{0.10}{12}}\right)^{72}\right\} 0.10/_{12} = 1,00,000$$

Or R x 53.978665 = Rs. 1,00,000 Or R = Rs.1853

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* Teaching note prepared by Prof. Shiladitya Roy of the Institute of Rural Management, Anand

The Deviations

However, the present value annuity formula used above, works only if interest is applied on the periodic reducing balance. Sometimes one comes across loans, which have equal periodic repayments but interest is not applied on the periodic reducing balance. Examples of such loans are those where interest is applied on flat rate basis or where interest is applied on a reduced balance of an earlier period (i.e., not on the balance of the immediately preceding period). These situations are discussed below along with a comparison with the standard annuity application.

Interest on Flat Rate Basis

Consider a loan of Rs.1000/- bearing an interest of 12% per annum – to be repaid in 12 equal monthly instalments. If interest is to be charged on flat rate basis, then it is a flat 12% on Rs.1000/- i.e., Rs.120/-. Hence, on flat rate basis each equal monthly instalment would be Rs. 1120/12 = Rs.93.33. To find out the effective monthly interest rate that is being charged one needs to determine the IRR of this cash flow stream – i.e., an inflow of Rs.1000/- in time '0' followed by 12 equal outflows of Rs.93.33. If calculated, the IRR would work out to 1.788% implying an effective monthly interest rate of the same quantum. The nominal annual interest rate would thus be, 1.788% x 12 = 21.456%. **Thus, this flat rate interest charge of 12% per annum, translates to an annual interest rate of 21.456%.** If, on the other hand, the above scheme was to charge interest on monthly reducing balance, then the equal monthly instalment, say 'R' can be determined by the annuity formula:

$$R\left\{1-\left(\frac{1}{1+0.01}\right)^{12}\right\}/_{0.01} = Rs.1.000$$

or R x 11.25508 = Rs. 1.000

or R = Rs.88.85

As would be logical, the IRR of the consequent cash flow stream would be 1% i.e., 1% per month or 12% per annum.

Equal Monthly Instalment But Interest on Annual Reducing Balance

Consider a loan of Rs. 10.000/- bearing an interest of 12% per annum – to be repaid in 24 equal monthly instalments. It the above scheme is to charge interest on monthly reducing balance, then the equal monthly instalment say 'R' can be determined by the annuity formula.

$$R\left\{1-\left(\frac{1}{1+0.01}\right)^{24}\right\}/_{0.01} = Rs.10.000$$

Or R x 21.24339 = Rs. 10.000

Or R = $\frac{10.000}{21.24339}$ = Rs. 470.7347

The effective monthly interest, in this case, as is logical would be 1%, which can be verified by determining the IRR of a stream of cash flows – consisting of an initial inflow of Rs.10.000/- followed by 24 equal outflows of Rs.470.7347. The IRR would come to 1% implying a nominal annual interest rate of 12% per annum. One however, can come across situations where, while the repayments are on equal monthly instalments, the interest is not applied on monthly reducing balance but on the balance of an earlier period. For instance, consider a situation where for the above loan, interest is charged on annual reducing balance. This situation gives rise to two questions as under:

- What would be the equal monthly repayment instalment?
- What is the effective interest rate?

Coming to the first question i.e., determination of the equal monthly instalment, it is obvious that the standard annuity formula would not be applicable here since interest is not on the periodic reducing balance. To determine the instalment amount, one has to take recourse to simple algebra. If 'x' is the equal monthly instalment, then the total amount paid through the 24 instalments is Rs. 'x' x 24. This should cover the principal loan amount of Rs. 10.000/- plus the interest charged in the two years. Since, the interest is on annual reducing balance, the interest for the first year is Rs.10.000 x 0.12 = Rs. 1200/-. The interest for the second year would be 12% of the opening loan balance of the second year. The opening loan balance can be calculated as under:

Opening balance –first year	:	Rs.10.000/-
Add: interest on the first year's opening balance	:	Rs. 1.200/-
	:	Rs.11.200/-
Less: total instalment amount of first year	:	12x
Opening balance, second year	:	11.200 – 12x

Thus,

x x 24 = 11.200 + (11.200 – 12x) 0.12

Solving the above equation, one gets the value of 'x' which is the equal monthly instalment, which comes to

x = Rs.493.08

Coming to the effective interest rate, it is obvious that it would be higher than 1% per month since the instalments are monthly but interest is charged on the opening balance of the year. The IRR of the stream of cash flows – initial inflow of Rs. 10.000/- followed by 24 monthly outflows of Rs. 493.08 – comes to 1.393%. **Thus the effective monthly interest rate comes to 1.393 % involving a nominal annual interest rate of 16.72 and not 12%.**